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Dilation of Granular Materials under Confining Forces

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Abstract

To study the compaction and dilation behaviors of a granular system, a new fiber optic displacement sensor was explored and used to detect bead pack motions during repeated vertical tapping. We investigated the limitations of the new sensor and found ways of optimizing its effectiveness in current and future experiments. We studied how dilation changes with the amount of confinement and the materials' packing fraction with a goal of finding effective ways to control the dilation and observe compaction under controlled dilation.

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Thesis Title: Dilation of Granular Materials under Confining Forces

LAKE FOREST COLLEGE
Senior Thesis

Dilation of Granular Materials under Confining Forces

by

Auriana A. Johnson

April 23, 2014

The report of the investigation undertaken as a
Senior Thesis, to carry two courses of credit in
the Department of Physics

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Abstract

To study the compaction and dilation behaviors of a granular system, a new fiber optic displacement sensor was explored and used to detect bead pack motions during repeated vertical tapping. We investigated the limitations of the new sensor and found ways of optimizing its effectiveness in current and future experiments. We studied how dilation changes with the amount of confinement and the materials' packing fraction with a goal of finding effective ways to control the dilation and observe compaction under controlled dilation.

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I. INTRODUCTION

People encounter granular materials on a daily basis and yet, most do not know how to define what constitutes such a material. There are many examples including sand, grains, beads, or even children's LEGO toys, but what makes these objects fall under the category of granular materials? A granular material is a large collection of solid, *macroscopic* particles that make up an *athermal* system. A major difference between granular materials and solids, liquids, and gases is that the particles, or grains, that make up this material are large enough to be seen by the naked eye. Because the grains are so large, the gravitational energy of a single grain is much larger than its thermal energy. This means that any thermal changes to the system will not have an effect on its behavior, making the typical statistics and laws found in thermodynamics inapplicable.

When listing examples of granular systems, it may be noted that all of the individual grains that make up the system are solids whose individual motions and interactions with the environment can be described using classical Newtonian physics. In fact, many introductory physics courses use spherical objects to illustrate and simplify ideas needed in building a good foundation in the discipline. However, the massive number of interactions that occur between these simple objects creates an extremely complicated system whose intricate properties still have physicists perplexed today.

An individual sphere rolling around a confined space is conceptually an easy problem to tackle but, as Benjamin Simpson (LFC '08) describes in his thesis entitled *Granular Compaction under a Confining Force*, adding more spheres into the system begins to quickly complicate the problem.¹ The behavior of a system with as few as ten spheres of similar size and shape would be difficult to predict because the boundaries and the surface are no longer the only main factors contributing to their trajectories

These objects are now frequently colliding with one another and throwing each other off their trajectories, sending them flying into other spheres or the walls causing the predictability of what happens to become mathematically very complex even under the most simplified and idealized conditions. In addition there is uncertainty in the true size and mass of each particle in the system and the inelastic collisions that occur cause a significant uncertainty in how much energy is lost. As these uncertainties begin to add up the initial conditions of the system can no longer give reliable predictions to the particles' positions and trajectories once everything is set in motion.¹

Knowing how complicated a ten ball system can be, imagine a system with thousands of these macroscopic particles all confined in a cylindrical container. If the container is held fixed, contact forces and friction dominate, keeping the granular material in a solid-like state that will remain unchanged unless an external force, a vertical tap for example, perturbs the system. Shaking the system allows the material to expand, diminishing the forces holding the grains in a steady configuration, and allowing them to rearrange in new ways. The expansion caused by shaking allows a majority of the material to leave its solid-like, metastable configuration and enter an amalgamation of all three possible "states". When the disturbance stops, the upper level of grains fall back down inelastically on top of the lower levels of grains. These behaviors create large amounts of uncertainties in the calculations required for predicting the material's behavior, making it extremely difficult to know exactly what each of these particles are going to do under certain sets of initial conditions. Unlike thermal systems, there is no set, agreed upon theory as to how these granular systems are supposed to behave as they are tapped and allowed to compact into new, metastable configurations.

Although the individual grains of a granular material are solids, as described above, the material itself does not solely behave as such. In fact, granular materials cannot be treated as a solid, liquid, or gas alone, but can exhibit properties that may make it appear to behave similarly to one or more of these states.² Resting in a container or in a pile on a stationary surface, the material appears to exhibit solid state properties. Unlike a solid, the internal pressure of a granular material is independent of filling height.² This is not to say that the pressure at the base of a granular material does not vary at all with the height in which it raises, but after a certain height has been achieved, the pressure reaches a maximum as any additional force caused by upper levels of grains transfers to the boundaries of a container. This is why an hourglass filled with sand is a reasonable method for accurately tracking the passage of time and an hourglass filled with water would not be. Unlike water, the pressure near the base stays reasonably constant as the amount of material varies in the upper container, allowing it to flow through the opening at a nearly constant rate. Water has historically been used to track the passage of time, but the rate at which it flows through the opening decreases as more water flows into the bottom container making it more complicated to accurately measure time.

Like a liquid, granular materials possess the ability to exhibit flow-like behaviors. A granular material will remain in a solid-like state when left unperturbed. This solid-like state is a metastable configuration, meaning if it is influenced by an outside force the stability of that configuration is lost. If the surface supporting the material is tilted past the angle of repose or the material is subjected to a vibration, the grains will begin to slide past each other, or flow. The grains will continue to flow until they reach a new equilibrium with a tilted surface or the external disturbances stop.

Gases consist of molecules that are constantly moving in quick, random motions causing frequent, elastic collisions. Unless undergoing a collision, these molecules do not significantly affect the behavior of other molecules in the system. A granular material subjected to high energy disturbances does exhibit gas-like properties; however, collisions experienced by the grains are inelastic causing a fundamental difference between the two types of materials. For example, a single ping-pong ball thrown against the floor will bounce. The collision with the floor is not completely elastic but enough energy remains in the system to cause the ball to bounce up and down for a while until the collisions convert the energy in the system into thermal energy. Fifty ping-pong balls confined to a thin bag thrown against the floor will smack into the ground and stay there due to the large amount of the energy being converted into thermal energy. The inelastic nature of granular materials is why gravel is a more desirable surface than cement for playgrounds and also why running on the beach is harder to do than running on a track.

A better understanding of granular materials may lead to significant improvements in many industries. Although many people don't realize it, granular materials are one of the most manipulated materials in industry and are a staple in many of our most important industrial processes such as food processing, pharmaceuticals, and construction and yet, their behaviors are relatively unknown. There are estimates that industries waste approximately 40% of their capacity simply from the transport of granular materials.² If we could get a better grasp on how granular materials shift and change during transport, their packing and storage could be drastically improved making many industries noticeably more efficient and cost effective.

II. THEORY

A. Compaction

Granular materials possess a unique ability to vary the overall density of the system while keeping the number of particles constant. This can be illustrated with a brand new box of cereal. Sitting down for breakfast, many find that when opening a new box of cereal there always seem to be a significant amount of empty space. Contradictory to popular belief, this is not merely the cereal factory's attempt to make the consumer pay more for less. The problem lies in the fact that cereal is a granular material that is transported in vehicles that shake. Fresh off the assembly line, the box was most likely completely filled but, as it was repeatedly shaken during transport, the pieces of cereal

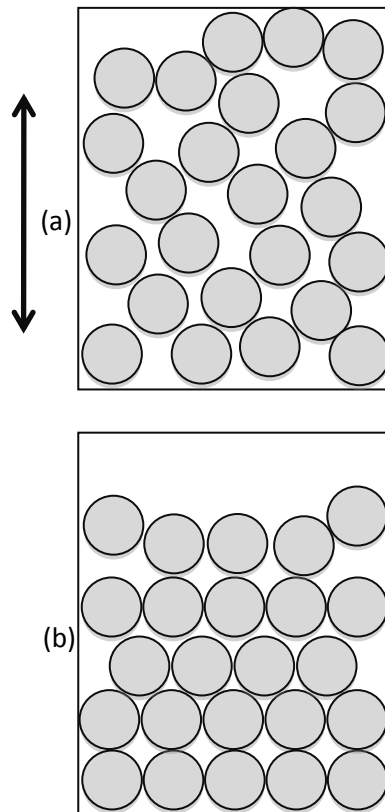


FIG. 1. (a) A granular system in a low density state. (b) The same granular system after being shaken vertically is now in a more dense state

were disturbed out of their metastable state and allowed to *compact*, illustrated in Fig.1.

The process of increasing a granular material's density is called compaction, which has attracted a lot of attention in recent years and has caused spirited debates amongst physicists studying granular phenomenon. The controversy is not over whether or not compaction occurs but is instead focused on *how* granular materials compact. The goal of studying compaction is to discover a reliable and agreed upon theory to describe and predict the behavior of granular materials exposed to external disturbances and to study compaction's dependence on the pack properties and the nature of the disturbance. However; even with the high level of interest, a consensus on the matter has not yet been reached.^{3,4}

In its loosest random packing state, the system, although mechanically stable enough to remain in a solid-like state, is not in its most structurally sound metastable configuration.⁵ Phillippe and Bideau explain that imposing an external perturbation on a granular system in an existing metastable state gives the system a sudden increase in mechanical energy⁴, allowing the material to escape its current metastable configuration and relax into a new one. This new configuration often has a lower energy than the previous state and can result in an increase in the material's density. This is a unique behavior that is found in granular materials that would not be expected to be found in a solid, liquid or gas. If a granular system is set up to fill a container to a particular height and is set to shake overnight, the next day that same system will occupy less space. Without changing the amount of the material in the container, its density has increased, and now occupies less volume. If the same experiment is done with water, for example, after the shaking has stopped, the next day the density would be the exact same as it was when it was initially setup.

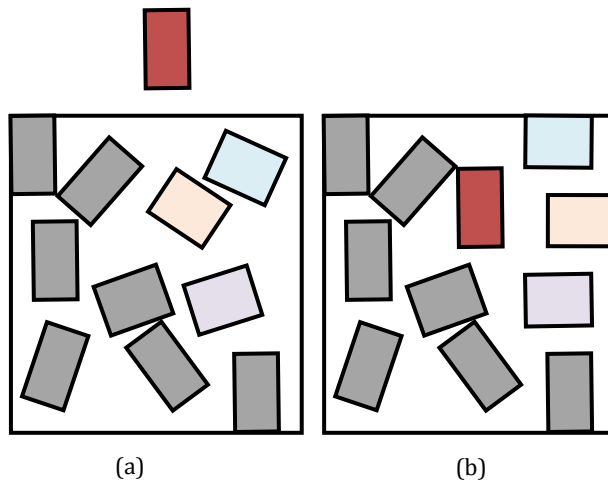


FIG. 2. (a) An unmarked parking lot with no room for a new (red) car.
 (b) The blue, orange, and purple cars have rearranged themselves and the red car can now fit into the lot boundaries

Jaeger uses cars in an unmarked parking lot to illustrate that even under the free surface conditions granular compaction is a very slow process.² If an unmarked parking lot is filled with randomly parked cars, a new car wanting to enter the lot requires the vehicles already occupying the space to rearrange. In this filled lot, there are free spaces in between the already parked vehicles but no single free space is large enough to fit an entirely new car. To find room, there needs to be cooperative motion of multiple cars. As more and more cars try and squeeze into this lot, more and more of the already parked vehicles are required to perform this cooperative motion to gather enough free space, but, as the spaces between parked cars become smaller, the harder it is to coordinate the movements to free up the correct amount of space. Fitting new cars into the lot becomes a lot more difficult or, at some point, impossible.

Allowing dilation, or the increase in volume of the material, gives the grains more space to try and coordinate the correct movements to allow compaction to occur. If the parked cars were allowed to move outside the boundaries of the lot momentarily, it would

make the rearrangement process a lot easier. Removing that dilation, while making these movements a lot more difficult and time consuming, does not eliminate compaction altogether. In other words, forcing already parked cars to remain in the lot reduces the amount of possible arrangements but does still allow movement as long as there is enough free volume already within the boundaries of the already occupied space.

B. Packing Fraction

A granular system contains both the material of the grains themselves and the space in between those grains. The total volume of the grains is given by their total mass divided by their density.

$$V_{grains} = \frac{m_{total}}{\rho_{grain}} \quad (1)$$

This is the total volume of material excluding the spaces found between grains. Because of the macroscopic nature of the particles, or *grains*, that make up a granular material, there will always be non-negligible gaps within the material which means the volume it occupies, $V_{container}$, will always be greater than the net volume, V_{grains} .

As the material is vibrated, V_{grains} does not change, but the gaps between the grains do, causing the total volume occupied by the granular pack to change. Taking the ratio between V_{grains} and $V_{container}$ describes how much of the container is actually being occupied by grains and how tightly the grains are packed in a volume. This is called the packing fraction of a granular material.

$$\phi = \frac{V_{grains}}{V_{container}} \quad (2)$$

Many physicists use the packing fraction of a granular material to describe its current density. Because of the unavoidable gaps between grains, ϕ will never be equal to one. Solid masses and liquids are examples of materials that will achieve a packing fraction equal to 1. The tightest or highest bulk packing fraction a granular material is able to achieve is when it is placed in a hexagonal close-packed or face-centered-cubic crystalline⁶ formation as shown in Fig. 3. These arrangements have packing fractions of⁷ 0.74. In this formation the gaps between grains is at a minimum and is a desirable configuration to optimize space; such configurations are commonly used such as when oranges are stacked at a grocery store. Without special care, however, this configuration will not often occur naturally.

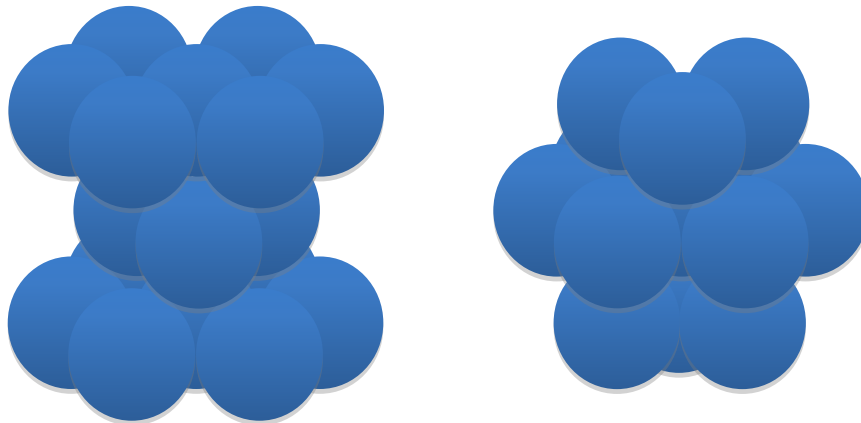


FIG. 3. A hexagonal close-packed crystalline structure and a face centered cubic crystalline structure. Figure on left is hcp. Figure on right is fcc.

Placing every individual grain into these crystalline forms would be very tedious and time consuming. Many of the granular systems encountered and manipulated on a daily basis are in a random state. Because the packing fraction is very unlikely to reach a fraction as high as 0.74 randomly, there must be a different maximum that a random system is able to achieve. This maximum, referred to as the random close packing, is

generally accepted to be near⁷ 0.64. At this point the grains are very close together and the gaps have become so small that grains are not able to reconfigure in a way that gathers enough free volume to allow the material to compact unless they crystallize. In practice a granular system can become partially crystallized, which often happens along flat walls. In these cases, the packing fraction could be between the 0.64 RCP value and the completely crystallized 0.74 limit.

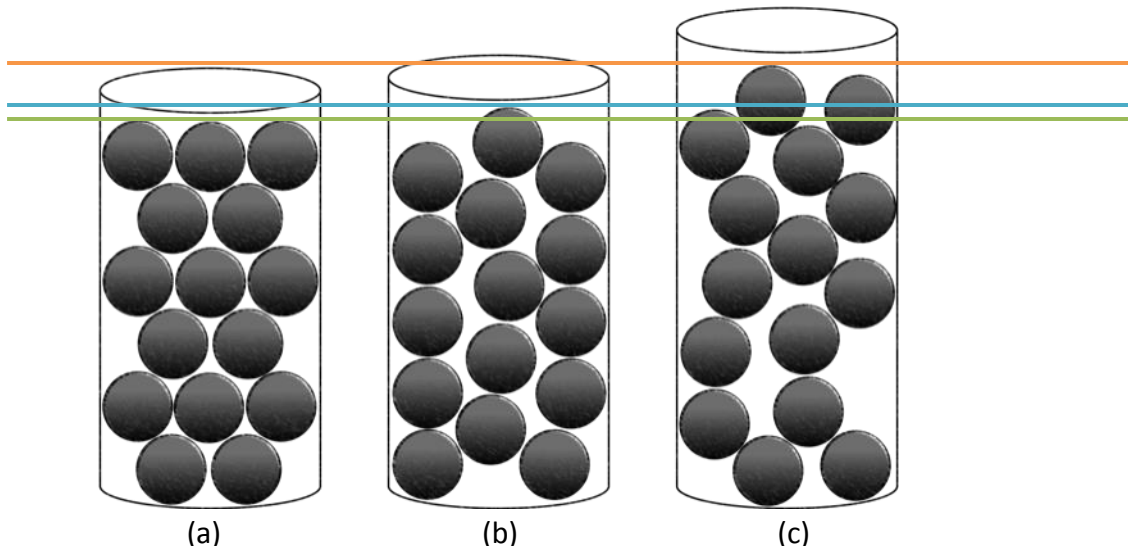


FIG. 4. The 3 figures above illustrate the difference between (a) crystallization, (b) random tight compaction and (c) random loose compaction. The heights of the packs, shown by the horizontal lines, differ but the number of beads in each scenario are the same.

There is also a lower limit to the packing fraction called the random loose packing. Onoda and Liniger suggests that at a packing fraction of 0.555 a granular material reaches its highest energy metastable state, or random loose packing limit.⁵ In other words, this is the loosest possible packing that is capable of remaining in a solid-like state. In practice, it is very unstable. Even the slightest disturbance would cause the material to collapse down to a higher packing fraction.

Our granular systems are contained in cylindrical tubes of known radius. By measuring the height of the granular pack, we are able to calculate the total volume of the granular system. Combining this with the volume of the grains, we can calculate the packing fraction of the system. To observe how the packing fraction of the system changes under vibrations we need only monitor the changes in pack height. If the container in which we place our material is an ideal cylinder, we can measure the radius and the height that the material occupies. Using those measurements we can then calculate the occupancy volume:

$$V_{container} = \pi r^2 h_{container} \quad (3)$$

To experimentally obtain the packing fraction of our system, we needed to be able to accurately measure the volume of the container being occupied. The height of the bead pack was estimated using the measuring tape attached to the container, but careful

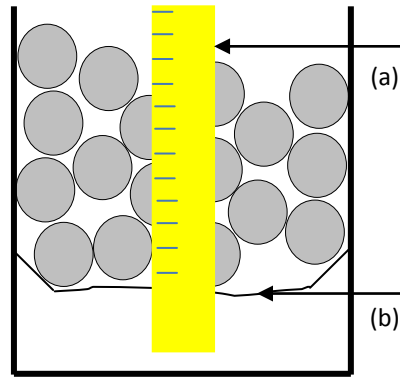


FIG. 5. (a) The ruler and (b) is the screen. Measuring tape was not always placed with an attempt to estimate the bottom of the measuring tape at 0.00 cm

analysis must account for the fact that our container was not perfectly cylindrical. The container was visibly misshapen by a screen placed at the bottom to allow for the flow of nitrogen gas used in previous fluffing techniques. Because of the inability to say exactly where the bottom of the container's surface was with respect to the ruler taped on to its side along with the oddly shaped screen at the bottom, a volume correction needed to be performed to get an accurate way of measuring the occupancy volume since using Eq. 2 would cause large inaccuracies resulting in an unreliable packing fraction.

To accomplish this, water was placed in the empty space between the bottom of the container and the top of the screen. A large buret filled with water was then securely fixed on top of the opening of the container and water was poured into the container at 10 mL intervals. After each addition of 10 mL of water, the height to which this amount rose was read off the ruler on the side. The data collected was used to plot the linear relationship seen in Fig. 6 between the volume of water and height of the tube occupied. The straight line found in Fig. 6 can be represented by the equation

$$V_{\text{container}} = \pi r^2 h_{\text{container}} + V_0 \quad (4)$$

where πr^2 is the slope of the line and V_0 is the offset volume or the volume that could not be found by simply relying on the ruler from the container. By doing this correction, the error in the $V_{\text{container}}$ measurement is drastically reduced and will now be mainly influenced by the uncertainty in height measurements.

Achieving a random loose packing in an easy, reproducible way is an important

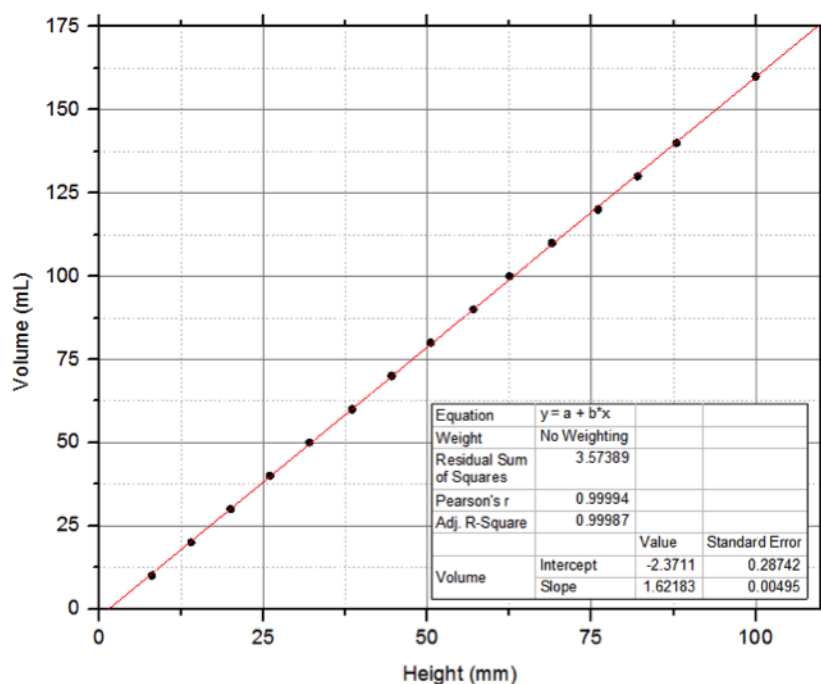


FIG. 6. Data from the volume correction performed on one of the tubes and its scale. Because rulers were hand placed, they varied from tube to tube and therefore each tube required its own correction

part of the initial setup. Many granular experiments achieve an initial loose packing by flowing nitrogen gas up through the bottom of their packs. In our system, we found this method to achieve a consistent packing fraction of 0.551; however, we suspect that there are inhomogeneities in the pack due to the sudden collapse of the system when the supply to the gas is shut off and the beads fall into place. We proposed and tested three new ideas to see if we could get a similar and consistent value for the loose state packing while improving upon the pack's consistency throughout.

The first method tested was placing a plunger with four holes of equal size and shape at the bottom of the empty container. Each opening in the plunger had a surface

area capable of holding approximately eleven beads and was about four bead diameters deep. The beads were then poured into the container and the plunger was slowly pulled up through the pack. By doing this as slowly and as evenly as possible, the individual grains that fell through the plunger were allowed to stabilize before additional grains hit them from above. We also tried rotating the plunger head while pulling it up through the pack, but making sure the plunger was not disturbing the already fallen beads was difficult and resulted in a higher packing fraction. We then tried slowly pouring the beads in the tube while pulling the plunger straight out. This required two people to do and caused the packing fraction to increase slightly compared to just pulling the plunger straight through when the bead pack was already in the container.

The second and third methods were closely related to one another. In each method a tube with a much smaller radius than the container was placed inside the empty cylinder. In the second method, the beads were poured only in between the inside walls of the container and the outside walls of the smaller tube. In the third case beads were poured both inside and outside the smaller tube so that the height of the two sections of beads were approximately equal. The inner tube was then slowly pulled up through the pack and the beads were allowed to settle. Both of these techniques gave the illusion of producing very low packing fraction but it was observed that the top surface of the bead pack was extremely uneven. Doing either of these created either a mound in the middle of the surface or beads clung to the walls of the container. This allowed the measuring surface to be held up by only a few beads and did not make contact with a majority of the surface beads resulting in a lower packing fraction than the system was likely able to achieve.

TABLE 1 Different techniques attempted and the average packing fraction and the reproducibility of each method

<i><u>Techniques</u></i>	<i><u>Average Packing Fraction</u></i>	<i><u>Standard Deviation</u></i>
air	0.551	0.004
plunger (straight)	0.554	0.002
plunger (rotate)	0.567	0.006
plunger (pour and straight)	0.556	0.001
inside skinny tube	0.553	0.002
outside skinny tube	0.551	0.002
inside/outside skinny tube	0.550	0.003

Each method was tested five times and their results were averaged in order to get their packing fractions. As seen in Table 1, all three methods give similar packing fraction values; however the plunger simplified our initial setup and was on par with the nitrogen gas method. Although with large amounts of beads the plunger needed a few strong taps to allow it to start moving upward, this method was suspected of also creating the most consistent packing fraction throughout the material and was therefore used in the initial setup throughout the trials found in this paper.

C. Dilation

For compaction to occur, the system needs to be perturbed out of its metastable state but another important factor in compaction is the idea that there needs to be a certain amount of free volume available to allow the grains to move around each other and reconfigure. If the current metastable configuration of a material is loose enough, gaps caused by bridging may simply be able to collapse which will increase the material's density. However, if the material has already undergone some compaction, the movement of the grains becomes very limited due to the decreasing gap size. The necessary free volume must be created by expanding. In 1885, Reynolds wrote a paper on

his notion of dilatancy. Reynolds says that, in order for a compacted granular material to undergo shear it must be able to expand.⁸ More simply, there must be free volume to allow grains to push and flow past each other. If there is not enough free space in the gaps between the grains of a material, the material must expand, or dilate, to allow grains to move and reconfigure. Although the idea that grains need space in order to rearrange may seem simple enough, in practice it almost seems counterintuitive as illustrated in a common observation.

If a sponge is saturated with water and is stepped on, the water comes pouring out. When taking a walk on the beach, many people notice that they leave dry footprints in the wet sand but, because this is such a common occurrence, the strangeness of this behavior is often overlooked. Putting pressure on a granular material's surface causes the grains on the surface layer to push against their neighboring grains which then push on their neighbors and on and on. The act of pushing other grains out of the way causes the gaps between the grains to increase. This increase in the gaps causes the amount of free volume to increase and the water flows down to fill these voids leaving the sand on the upper levels relatively dry. This effect, of course, will only remain while pressure is being applied. When you step off of the sand, the grains are able to compact again and the gaps decrease, forcing the water back up towards the surface.

D. Dilation confinement

Allowing a granular material to dilate strengthens its ability to compact. Dilation gives the material the ability to create free volume when the gaps between the grains start to become smaller and smaller. This expansion gives the grains freedom to move and reconfigure so that the material is capable of compacting. Although dilation improves the materials ability to compact, not all configurations of a granular material need dilation to experience changes in density. Even though the additional free volume is not needed for loosely packed materials, given a unconfined surface the granular material will still dilate.

Restricting or eliminating dilation of a granular material will reduce the rate with which it compacts and can lower the maximum packing fraction it is able to compact to.⁹ When exposed to an external disturbance, the large gaps caused by the bridging of grains begin to collapse, allowing the material to compact even though dilation has been completely eliminated. The restriction of dilation becomes more and more of a problem when the gaps between grains grow smaller. As the material becomes denser, there is a lot less free volume available to maneuver.

III. METHODS AND APPARATUS

A. Experimental apparatus

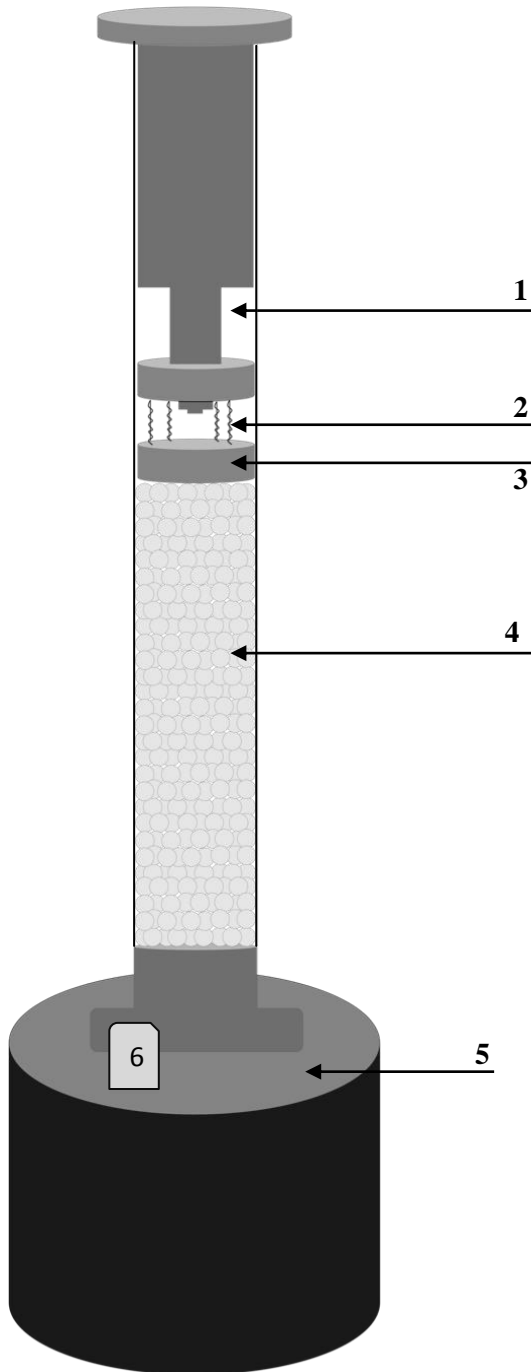


FIG. 7. A diagram of the experimental setup.
(1) Philtec displacement sensor and handmade holder.
(2) Spacers with springs to apply confining force. (3) Surface used to measure the packing height. (4) acrylic beads and tube. (5) Electromagnetic shaker that shook the entire system up and down. (6) accelerometer attached to electromagnetic shaker

Our granular material consisted of 1/8 inch diameter, spherical, acrylic beads placed in a long plastic cylinder. To stream line the setup process, beads from previous trials were vacuumed out of the cylinder. Doing this did create some static within the material. To reduce the effects of static, ionized nitrogen gas was flowed through the material and the interior walls of the cylinder were wiped down with an anti-static solution.

Once the anti-static step was complete, a measured amount of beads were poured into the cylinder on top of the plunger designed to “fluff” the beads. To get the plunger to start moving up through the material a few taps need to be given to the system. The VTS Model 100-6 electromagnetic shaker at the base of the setup, as seen in Fig. 7, is controlled by receiving a single cycle sine wave created by an SRS Model DS345

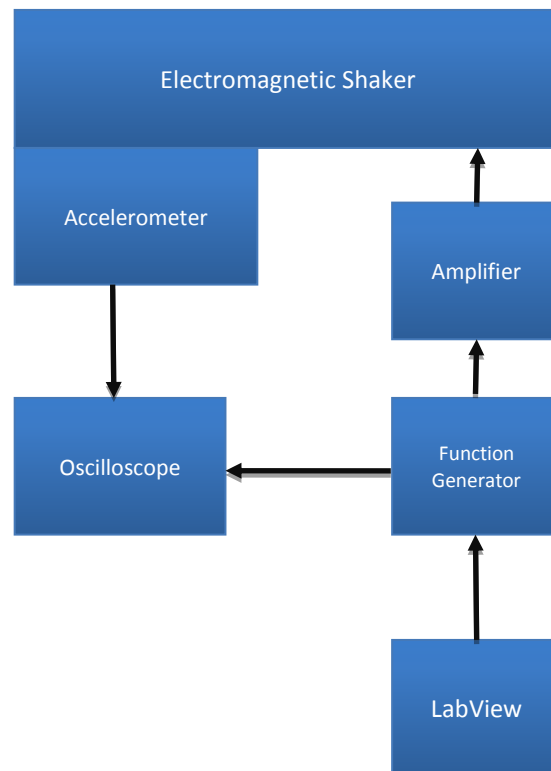


FIG. 8. Schematic of the connections for the experimental setup

function generator and amplified by the Crown CE100 amplifier. For all experiments being discussed, we used a 30 Hz sine wave with a 1 volt peak to peak amplitude which resulted in a maximum tap acceleration of roughly 2.0g, or 2.0 times the acceleration of gravity.

When the plunger has been pulled through the entirety of the material to create the initial loose packing, the Philtec fiber optic displacement sensor is secured to the top of the container. In Fig. 7, arrows 1, 2, and 3 are all part of the apparatus fabricated by Lake Forest College machinist, Aco Petrusevski. The holder, Fig. 7(1), is made to fit snugly and securely on the top of the container. It is also responsible for holding the delicate sensor in a fixed location relative to the container. The top and bottom surfaces are made of aluminum. The top surface is fixed to the bottom of the sensor's holder and attaches to the bottom surface, Fig. 7(3), with four screws and springs. The bottom surface is used to rest on top of the granular pack and allows us to determine the pack's displacement from the sensor over time. The springs and screws holding these two surfaces together are in parallel and have nominally equal spring constants of 2.45 lb/in.

B. Experimental procedure

With this set up, the goal was to be able to observe compaction and dilation behaviors of a repeatedly tapped system and to be able to control the dilation. To do this we implemented the use of a new fiber optic displacement sensor from Philtec. This sensor is made up of optical fibers that transmit and receive light. It sends out light with an intensity that is controllable through the Philtec LabView program. The light reflects off a surface and the reflected intensity is measured by the sensor which, through a manufactures' calibration, is converted into a displacement. To observe compaction and

dilation and to monitor the compression of the springs in the system, the Philtec LabView program for a data stream could be used. However, if packing fraction was to be measured, we needed a way of determining the pack height. In Eq. 4, the height of the bead pack is needed in order to calculate the occupied volume. Using the sensor's ability to detect its displacement from the surface resting on top of the pack seen in Fig. 7(3), we needed to be able to convert this into the corresponding height read from the side of the container. Before data collection could be accurately done, certain properties and limitations of the new sensor had to be explored to insure we were optimizing its use and were receiving reliable data.

This fiber optic displacement sensor is very sensitive and capable of reading displacements with high levels of accuracy. However, it does have limiting factors that needed to be optimized to make it consistent and reliable. The sensor comes with a premade LabView program and a manual that gives a basic overview of which operating settings should be used. Through experimentation, important limitations and quirks were discovered that had the potential of creating a significant amount of error in the calculation for the occupied volume and, therefore, the packing fraction.

Before we could begin calibrating the sensor to convert its displacement into a height or to even look at data streams showing the movement of the pack, we had to make sure that what the sensor was seeing and reporting was an accurate representation of what was happening.

To calculate the packing fraction of the material, the displacement reading needed to be converted into a height so that Eq. 4 could be used to find the occupied volume which, by using Eq. 3, could be used to find the packing fraction. Assuming the sensor is operating correctly, the change in displacement should be equal to the magnitude of the change in the bead pack's height since the bottom surface is simply shifting down over time. Experimentally this was not the case.

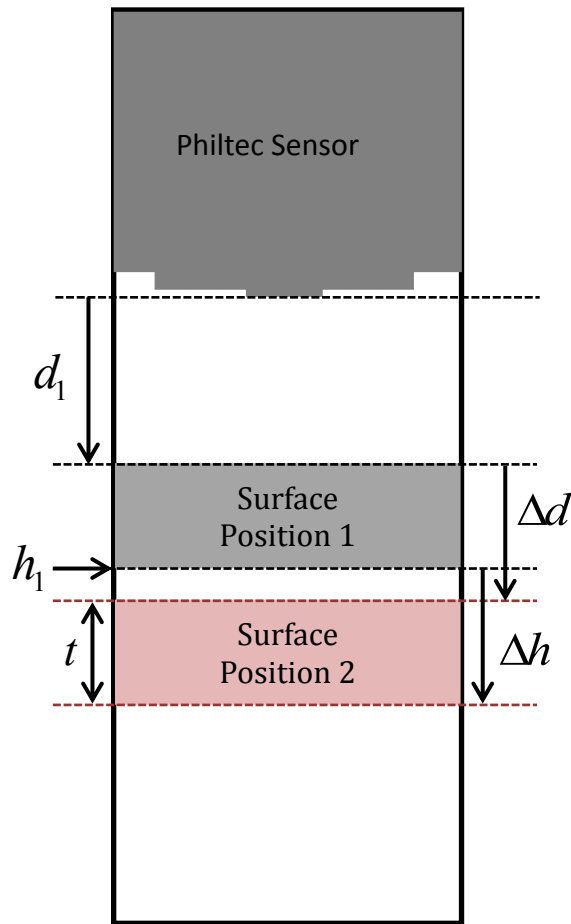


FIG. 9. Illustration of the bottom surface moving. Rigid nature of bottom surface dictates that Δd should be equal and opposite of Δh

A calibration program was created in LabView that would read the displacement from the top of the surface to the sensor. It asked the user to input the height of the pack read

from the ruler on the container and would then compact the system by exposing it to five strong taps. This would repeat for the number of calibration points the user wanted. The collected data could then be plotted, ideally resulting in a straight fit line fit with a slope of negative one.

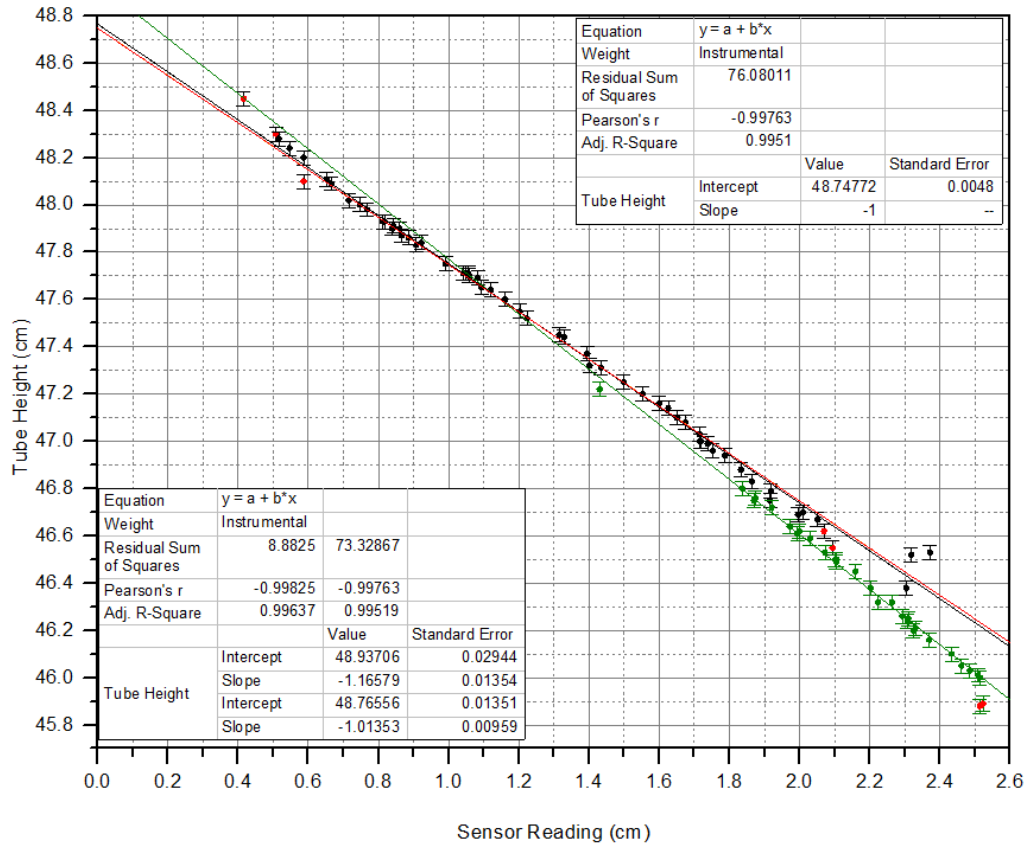


FIG. 10. Bad calibration. Different colors are results from different runs. The red line shows a fit with a forced slope of -1 which clearly does not fit the green data very well

Not only was the ratio between the change in height and the change in displacement not one, but, allowing the slope to differ from -1, the data did not fit well to a linear curve, which is illustrated in Fig. 10. Because the bottom surface is a physical object whose thickness (t in Fig. 9) is constant, this means the sensor's displacement value was not varying linearly as the measuring surface shifted. This sensor was built for the

purpose of determining the displacement between itself and an object with high accuracy; therefore, either something was wrong with the sensor's built in calibration, or it was operating under inappropriate operating settings. This was most likely an operator's error and so began the investigation of how different settings affected the displacement measurement of a stationary object.

One property of the sensor that needed attention was its sensitivity to environmental movement and lighting. With the sensor in position and the room lights on, the sensor was reacting to shadows cast about the room due to people moving around the lab. Trying to find ways of shielding the sensor from seeing movements unrelated to the bead

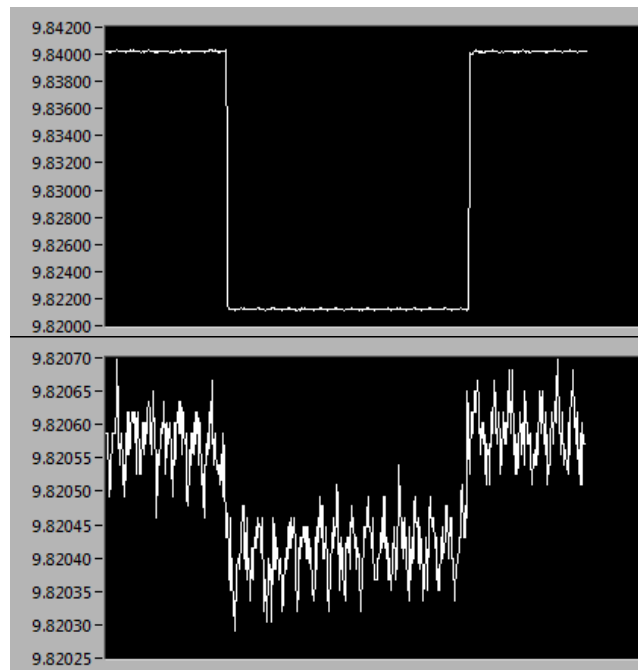


FIG. 11. When shutting of the lights, the displacement dropped (top). After covering the system in a tarp, the displacement jump drastically improved (bottom). Keeping the lights off also allowed movement around the lab without affecting the displacement reading.

pack, we tried shutting off the lights. No lights on meant no shadows could interfere with the sensor. When the lights were shut off, the displacement decreased by about 0.02 mm. This was much larger than fluctuations in the displacement reading caused by averaging and, if a system was initially running in a lighted room and the lights were shut off for the night, it would cause sudden dips in the packing fraction. We attempted to correct for this behavior in the sensor by blanketing our entire system in an opaque, black tarp and keeping environmental lighting to a minimum. The bottom figure in Fig.11 shows the effect that switching the lights on and off had with the set up under the tarp. The effect is still visible, but is drastically reduced to a jump of about 0.00015 mm instead of 0.02 mm.

The Philtec manual came with descriptions of the different user input settings the sensor could operate under, but until some experimentation was done, it was unclear how these settings affected the displacement reading. In order to observe dilation, the fastest averaging rate of 1 sample per second was used. This allowed us to see real time data of how the pack was moving and made observing and measuring the dilation easy to accomplish.

After receiving interesting data for an experiment where the bottom surface got stuck in the container and was not moving down as the pack did, it became clear that the averaging rate created a behavior that mimicked banding behavior. Since the surface stayed in a very similar position for extremely large amounts of data points, it became obvious that the banding was not due to the surface shifting with the granular material but was instead a behavior caused by an artifact in the displacement sensor that resulted in a

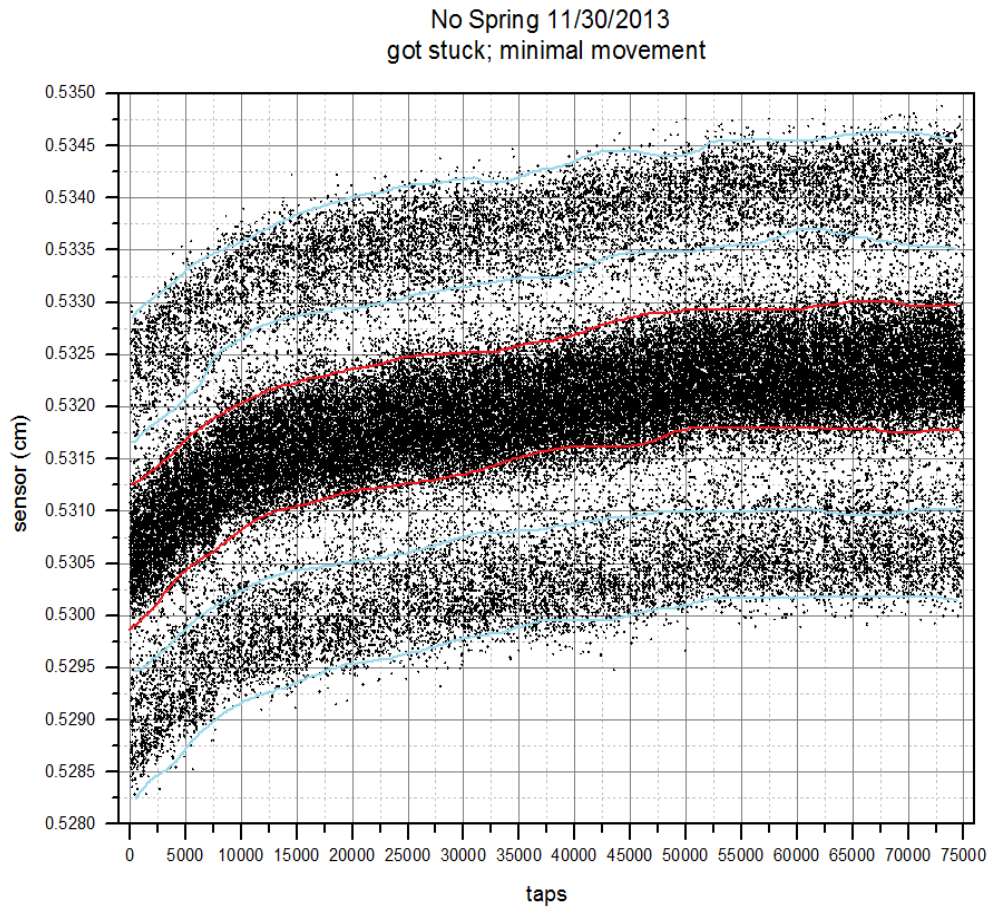


FIG. 12. The measuring surface had gotten stuck on the walls of the container and was unable to move down with the pack. The banding seen here is caused by the fluctuations in the displacement reading.

periodic variation in the reading. By setting the averaging higher, the fluctuations reduced from about 0.0095 mm to 0.00035 mm peak to peak as seen in Fig. 13. This could be reduced further by using the highest averaging, but this would drastically reduce the speed at which data was collected and there was concern of this slowing down the program created to calculate packing fraction which will be described in later sections.

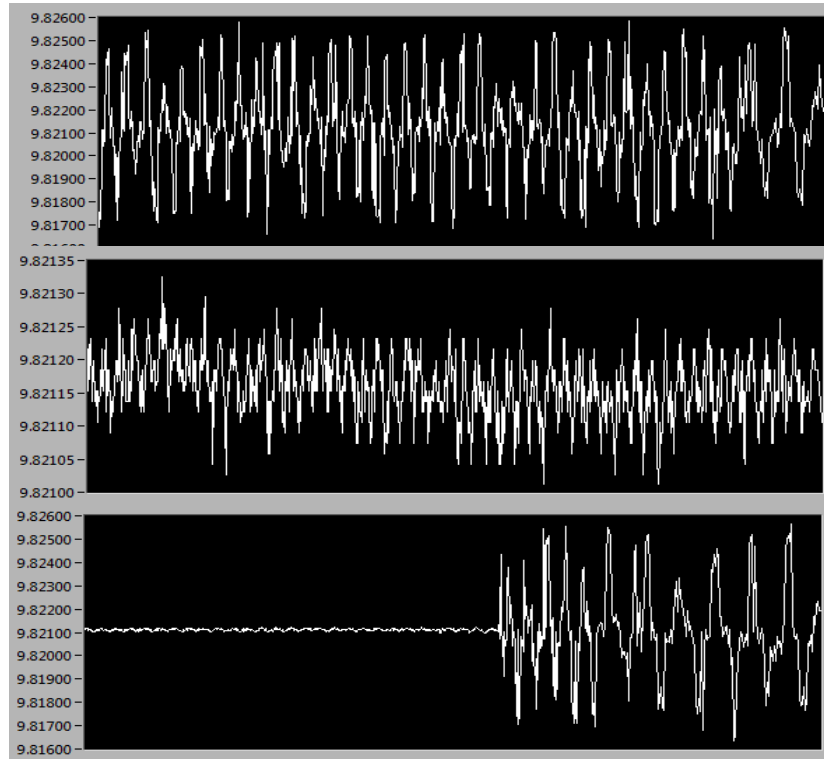


FIG. 13. Displacement readings for a stationary surface. Top graph is an averaging speed of 2 samples per data point. Middle graph is an averaging speed of 512 samples per data point. Bottom graph shows how they compare.

Even after finding these properties and implementing procedures to limit their effects, we were still getting bad calibration plots that conflicted with the fact that $\Delta d = -\Delta h$. By keeping the lights off during calibration, it was possible to move back and forth

between the computer and sensor without changing its displacement reading, and by increasing the averaging rate the variance of the displacement drastically decreased.

Another setting we were able to alter was optical power. As previously described, this sensor was made up of optical fibers that transmitted or received light. In the beginning the optical power transmitted was set to one hundred percent. This was found to be the source of the nonlinear displacement changes.

Because the sensor uses the reflected intensity to measure displacement, as the reflecting surface moved, the optical power received varied. What was happening with the transmitted power turned all the way up, was that the optical power received was reaching one hundred percent and oversaturating. This caused its calculated measurement of the displacement to become extremely unreliable. To fix this, the surface we use to measure the pack height was placed on a lab jack and the sensor was held above it using a ring stand and clamps. The height of the surface was varied and the optical power was lowered anytime the power received came close to one hundred percent. Eventually cycling through the detection range of the sensor we ended up using a transmitted power rating of 35 percent.

After this correct optical power rating and averaging values were found and the lights had been shut off, the system was now able to undergo calibration. Fig. 8 shows two of the calibration runs used for our current setup. After the first run, the sensor was taken off the container, the beads were taken out of the system, and the entire setup was reset. As the graph shows, each calibration fit very well with a forced slope of -1 and agreed on the maximum height, or the intercept, to five significant figures. Any shifting caused by resetting the setup was not significant enough to be noticeable; however taking the sensor out of its holder should be avoided. The likelihood of the sensor shifting up or down from the original position in the holder is very likely and will cause a vertical shift in the

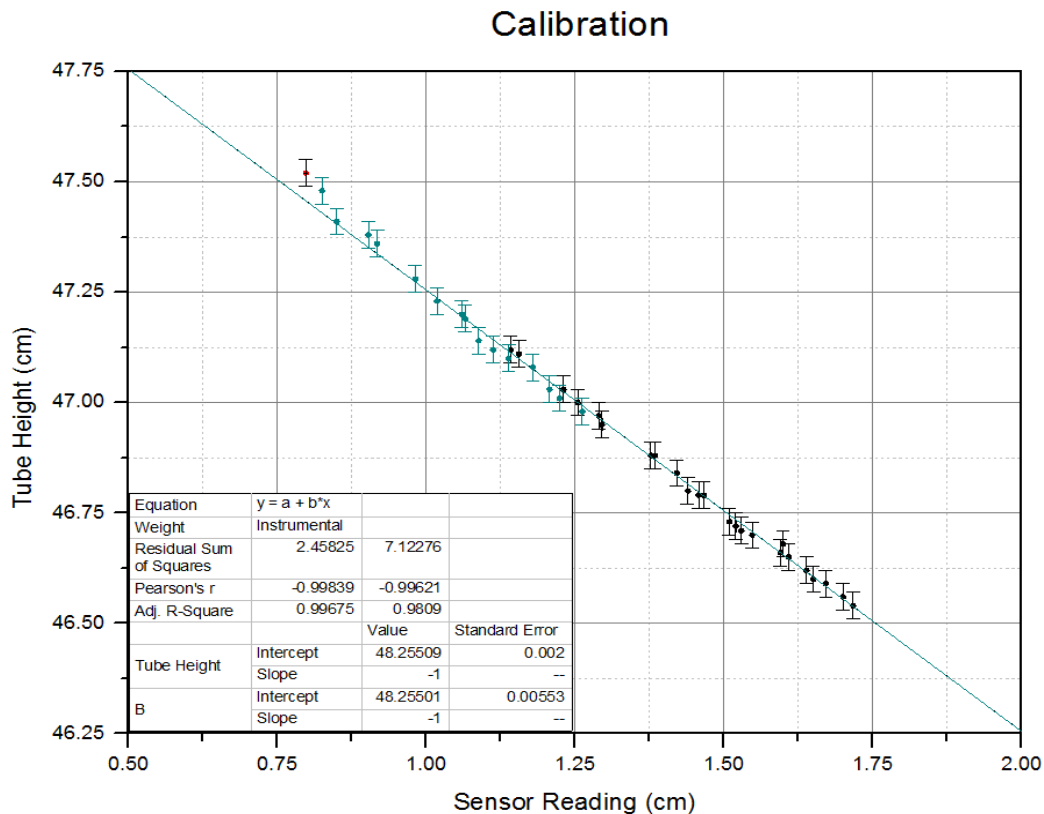


FIG. 14. Graph shows two separate calibration runs. Black data for run 1, blue data for run 2. As displacement from the surface decreases the calibration does not hold as well. This is most likely due to a limitation in the sensor and not a characteristic created by operating under non ideal settings

calibration.

From this calibration we can use the displacement sensor reading to calculate the pack height

$$h_{container} = -d_{sensor} + h_{max} \quad (5)$$

Hard wiring this equation along with Eq. 1, Eq. 3, and Eq. 4 into the code for the main collection program, we were able to use the displacement read from the sensor to calculate the packing fraction of the system. This main program allowed us to set the shaking parameters we wanted and the number of taps the system would undergo and, after clicking go, the entire setup is now completely automated.

The program was capable of reading the displacement received from the Philtec sensor and, using Eq. 5, turned that displacement into the height of the pack. The pack height, $h_{container}$, was then used in Eq. 4 to obtain the total volume occupied by the material, $V_{container}$. The density of the beads being used in these experiments was found to be 1.20 g/cm^3 which was used along with the total mass to calculate the volume of material, V_{grains} , using Eq. 1. Then, using Eq. 2, the main program calculates the packing fraction and then gives the system another tap. This process repeats as described above until the desired number of taps has been achieved.

IV. RESULTS

We were interested in observing compaction under controlled dilation. Applying a large compressive force to the bead pack's surface reduces dilation. We want to relate the spring compression to the dilation and determine if this dilation is independent of the current packing fraction.

As suggested in earlier sections, the amount of dilation dictates the rate of compaction. Our “free” surface model, where all the springs have been removed, is still not completely free to dilate. The surface used to measure displacement is a necessity for the sensor to be able to see the movement of the pack. This surface is made of shiny, smooth aluminum and rests on top of the pack so the sensor can detect the movement easily. Because we are unable to determine the dilation for this setup with truly zero confinement it cannot be said exactly how much the reflecting surface impedes on the material's ability to compact, it can however be said that it is not a large disturbance to an ideal set-up. It can be seen in Fig.15 that, even with this small amount of confinement, the system compacts very quickly in the beginning and has already reached a packing fraction of 0.61 by 13,000 taps. This is most likely due to the fact that, although the surface does provide a small amount of confinement, compared to a strongly confined system (Fig. 16), this “free” surface is allowed to dilate a relatively large amount (Fig. 17).

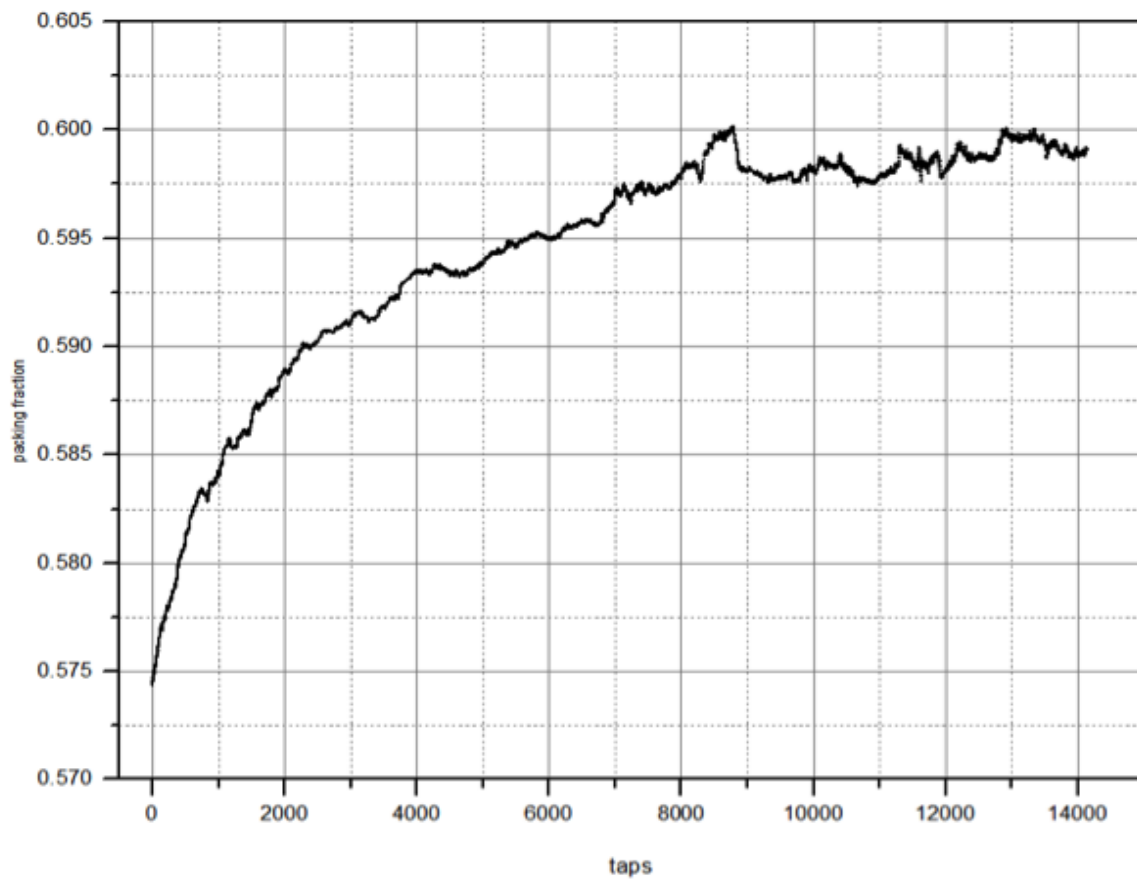


FIG. 15. A graph of a “free” surface compaction. The aluminum surface used to measure the height of the bead pack does have a certain weight, but does not create a significant enough force to significantly affect the materials ability to compact

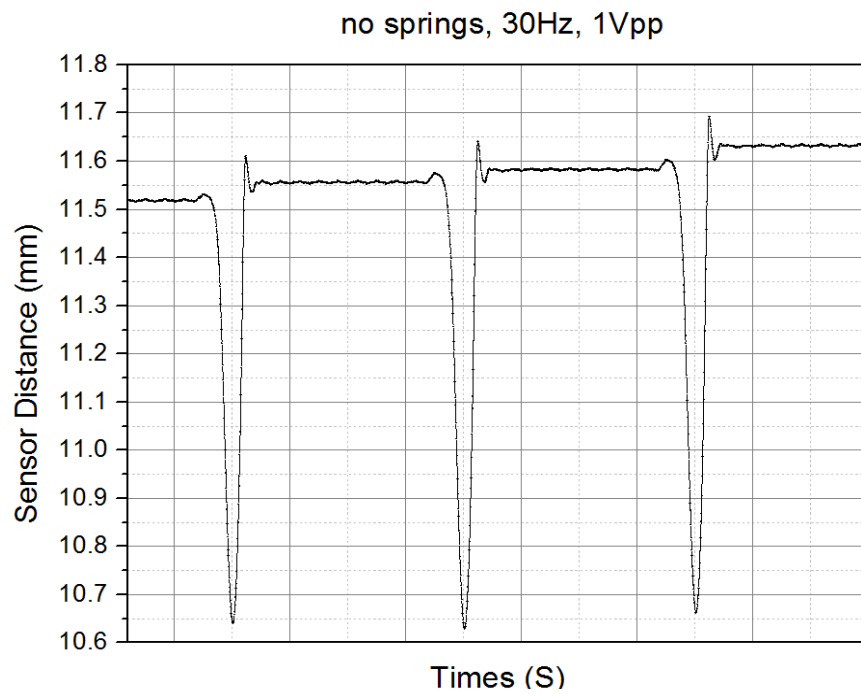


FIG. 16. Dilatation of our surface only under the confinement of the measuring surface.

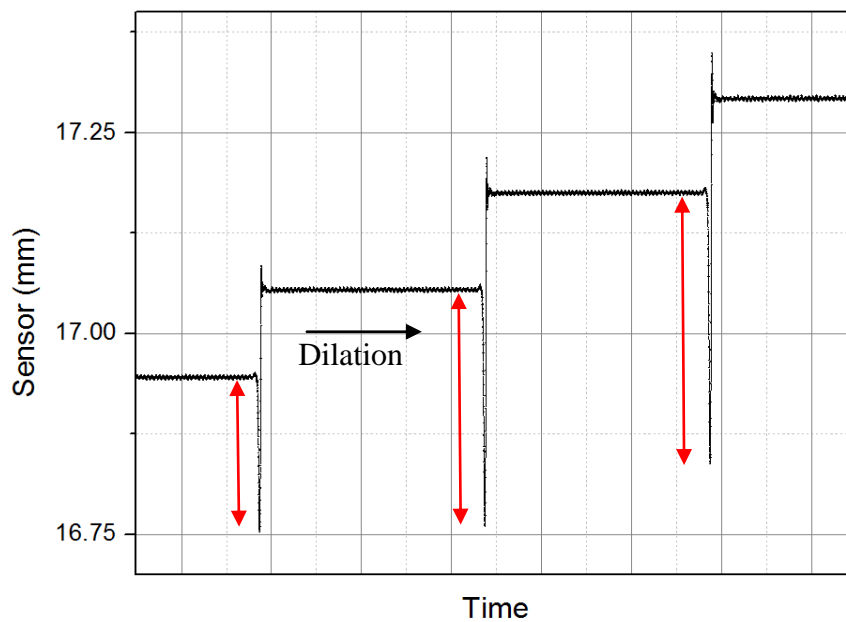


FIG. 17. Dilation under some confinement. Red arrows demonstrate the dilation measurement.

One of the main interests and focus in our experiments was trying to find a relationship between dilation and packing fraction and the spring compression or confining force. The confining force limits the amount of dilation the system can achieve. If the experiment calls for the dilation to be controlled but not eliminated, we need to know how much force to exert on the top of the pack to achieve the desired dilation. Data similar to what is seen in Fig. 16 and Fig. 17 was collected for varying bead pack masses and spring compressions. To measure the dilation that occurred during each tap, the minimum displacement found at the peak dilation was subtracted from the displacement found when the pack was at rest.

From Fig. 18, it appears as though there is some sort of definable relationship

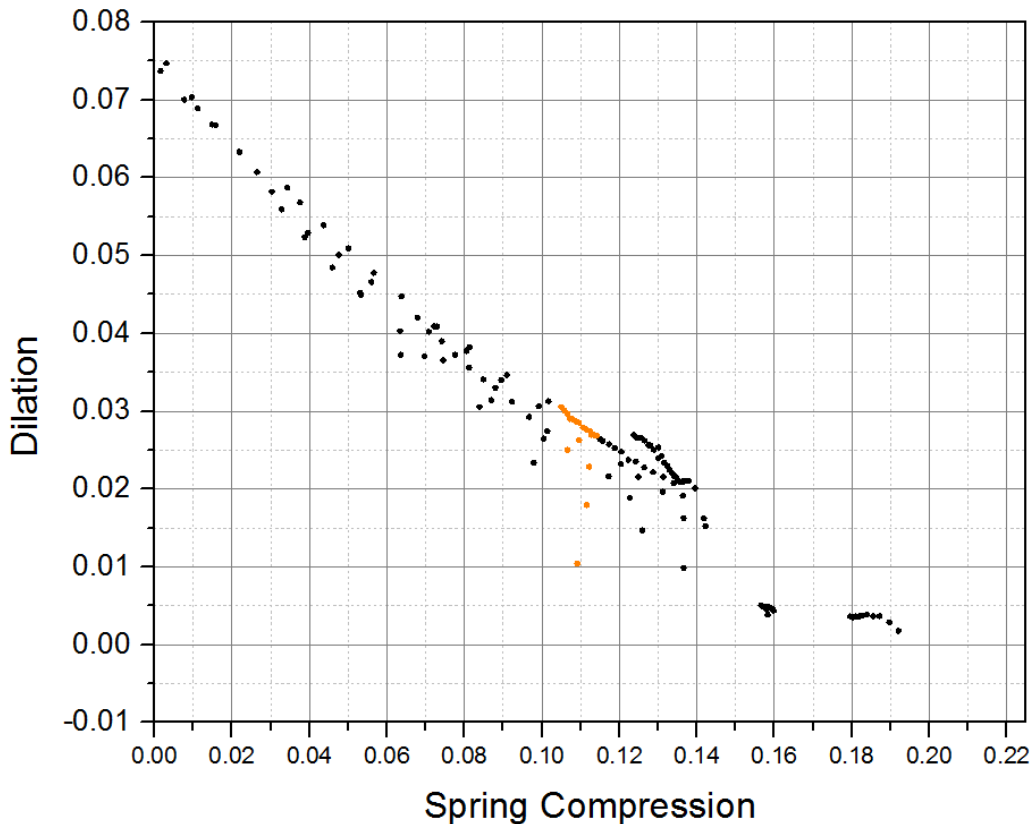


FIG. 18. A single run consisting of a bead mass varying from 503g-540g . Orange points will be seen in next graphs as well. These are the points where spring compression was at 0.11cm

between the amount of spring compression and the amount of dilation possible. It does not appear to be linear but instead starts at some, free volume maximum value of dilation and begins to decay down to zero as the confining force increases. As compression increases, the ability to detect a peak for the dilation in origin becomes increasingly difficult until the point where the noise from the averaging and the amount of dilation become indistinguishable. To keep the graphs readable, the compression was limited to about 0.2 cm.

This experiment was performed by starting with a system of 503 grams of beads that were put into a loose configuration. The maximum displacement of the sensor, where spring compression was zero, was found to be around 1.8 cm. This allowed the sensor to start about 1.67 cm away from the top of the bead pack. The system was then tapped about 20 times or until the maximum displacement had been reached. A few more grams of beads would then be added on top of the now slightly compacted pack in order to regain a distance from the top of the pack to the sensor near 1.67 cm. Not only did this allow for many data points per run but also allowed us to see if there was a possible relationship between dilation and the packing fraction of a material.

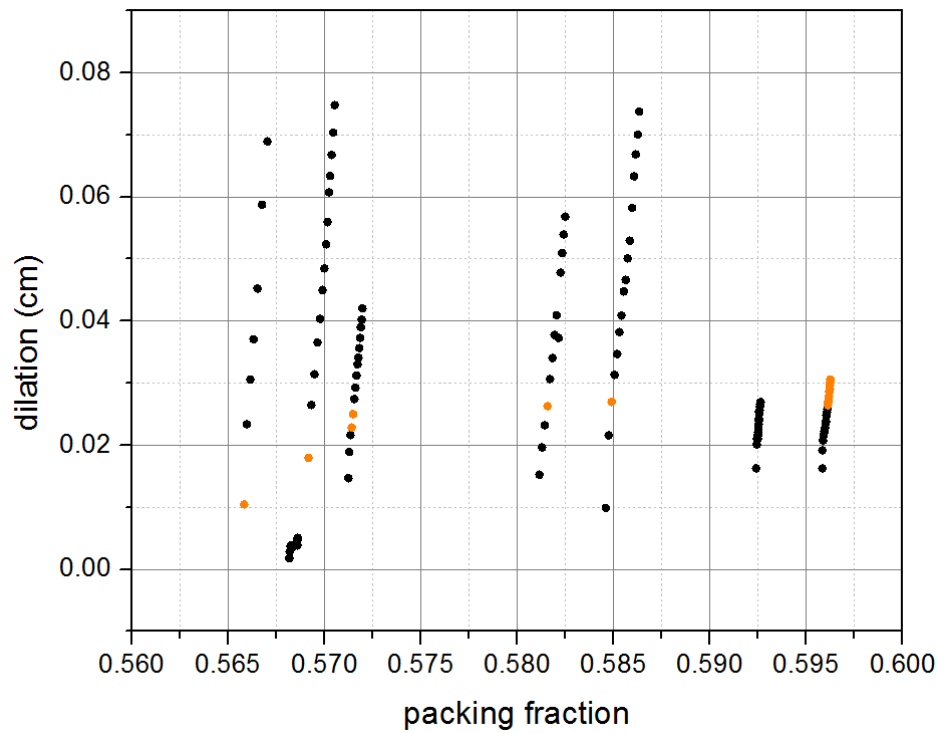


FIG. 19. This is the exact same data from Fig.18 but with dilation as a function of packing fraction. Points with spring compression of 0.11 have been highlighted.

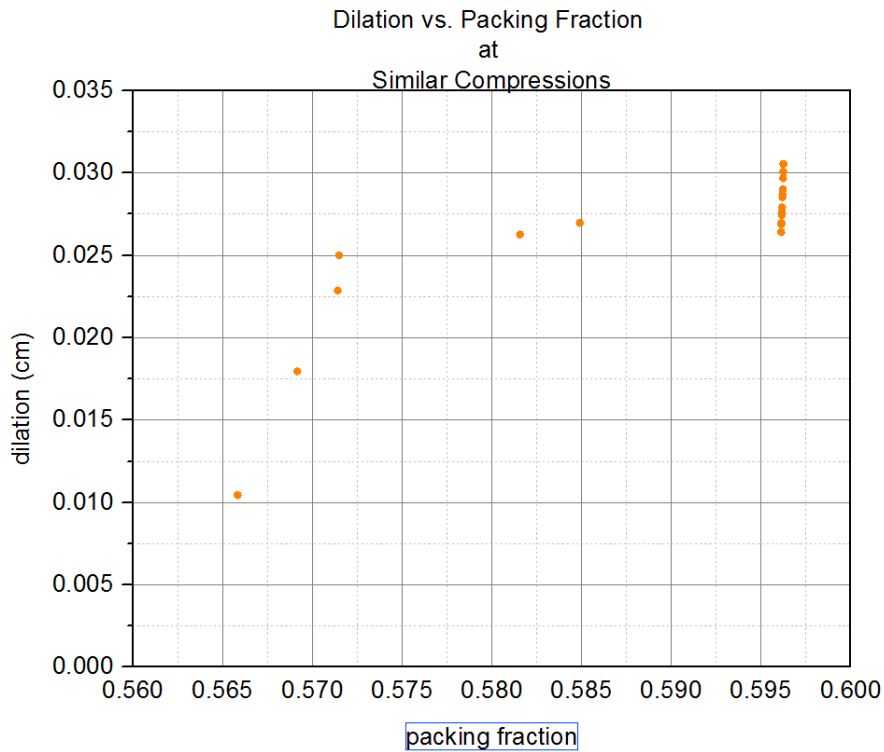


FIG. 20. This is the exact same data as Fig.19 with the points with similar spring compressions isolated.

Fig. 19 shows the same data as seen in Fig. 18 but with dilation as a function of packing fraction. Because in this particular comparison we need compression to remain constant, the data points that were recorded to have a spring compression of 0.11 cm to two significant figures were isolated (orange points seen in Fig. 19). These points were then plotted on their own, Fig. 20, to get a better visual of what is happening as the packing fraction of our material increases. It does appear that, as packing fraction increases, the amount of dilation increases as well. Although this is a very limited amount of data to be analyzing and the increase is very small, from what we know about granular materials' compaction behavior, this would be a logical behavior for the system to have. As a material becomes more and more dense, the amount of free volume begins to drastically decrease, which means, in order to compact, the material must dilate. It would make sense that the less free space readily available within the container between grains, the more that would need to be created by dilation. Until more of the data collected can be analyzed there cannot be much more discussion or exploration but the small amount of information we are able to look at suggests that the more compact a material is the more it will want to dilate keeping the confining force relatively constant.

Because we now have this fiber optic displacement sensor, the ability to search for the steady state packing fraction of a system whose dilation has been completely restricted is a possibility. Setting the large data collection program to run for an extremely large number of taps, the system will tap approximately once every two seconds and record the tap number, sensor displacement, tube height, optical power received, and packing fraction. As hinted at before, a spring compression any higher than about 0.2 cm makes dilation undetectable by the sensor. A system of 515 grams of beads was prepped resulting in an initial displacement of approximately 0.7 cm, or a

compression of about 1.1 cm. This experiment began on March 13, 2014, with a packing fraction of approximately 0.567 and as of April 23, 2014, has only reached a packing of 0.5704 after nearly a million taps. It is unclear if the system has reached its steady state or if it's even close to it at this point in time. The packing fraction is still, very slowly, increasing in steps of about 0.000001 every few hundred taps. This extremely slow compaction may suggest we could be getting closer to a steady state value, but the system will be allowed to continue for now as it is still increasing.

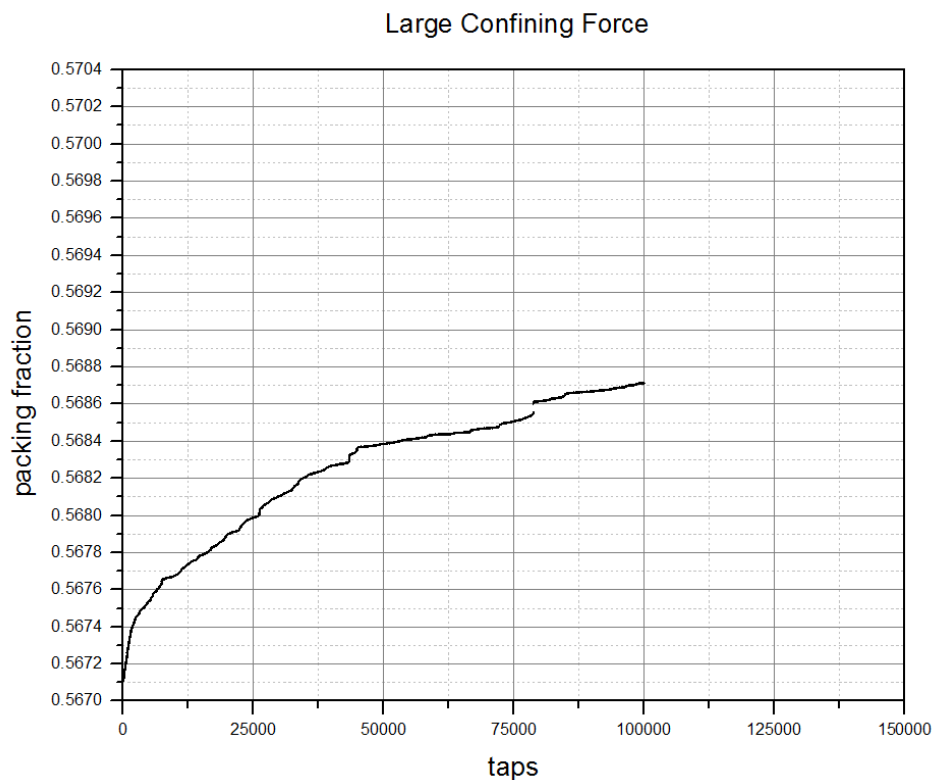


FIG. 21. Under extremely large confining for the system compacts very slowly. At this point in time the system has undergone nearly 1 million taps and has reached a packing fraction of about 0.5704. Due to the large size of the data files, transferring it over from Excel to Origin is temperamental and so far 100,000 taps are all that have been successfully transferred.

V. CONCLUSION

Through multiple experiments and investigations of the Philtec sensor's behavior, we were able to find settings that optimized its capabilities and write LabView programs to automate our setup. It is now possible to convert the displacement read by the sensor into a height, which can then be used to calculate the packing fraction of the material as it is tapped.

We are now able to obtain a large amount of data much more quickly than has been done in the past. Also, it is now possible to measure the dilation of the pack much more accurately with the Data Stream program from the Philtec Labview program. This will help collect quick and accurate data in future experiments. If the data stream program can be integrated with the main collection program, the system can become even more automated and all of the data needed can be collected at once.

One of the main interests and focal points of our experiments was trying to find a relationship between dilation and packing fraction and the spring compression. At this time in the experimental process, we have no way of exactly controlling the confining force being applied by the springs. An idea to obtain some control over the confining force to keep it at a similar value throughout a trial is to introduce spacers that could go between the sensor and the top of the container. As the pack shifts down, the compression of the springs lessens, causing the confining force to decrease. If the sensor started with a certain amount of spacers of known thickness between it and the container, the spring compression could be monitored by the sensor's displacement and when the pack reached a certain height that strayed too far from the desired confinement force, a

spacer could be removed, a new calibration could be put in place, and the trial could continue to run under similar conditions.

To implement the spacers, we need to know how much the compression can vary before the dilation suppression significantly changes, therefore a better understanding of the relationships between compression, packing fraction, and dilation is required. There appears to be a hint of a possible relationship between dilation and spring compression and also between dilation and packing fraction. The relationships that seem to be forming do seem to be logical behaviors. Increasing the amount of pressure put on top of a granular material should diminish its ability to expand. If the confining force remains constant, as the packing fraction increases so should the dilation. This follows Reynolds' idea of dilatancy; compacted materials need free volume in order to undergo shear. If there is no space within the system for the disturbed material to occupy, it must be created.

Completely restricting dilation from occurring does not eliminate a granular material's ability to compact but, as seen in the steady state experiment, it drastically slows down the rate at which the material is able to compact and also, because the material cannot expand, the steady state packing fraction may be reduced.

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